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The charge radius of a D$p$brane

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Abstract: Sen has shown that tachyon condensation in Dbrane–anti-Dbrane configurations can lead to remarkable connections between string theories. A consequence of his results is that there is a minimal value of the radial coordinate $r_c \propto \sqrt{\alpha'}(gN)^{1/(9-p)}$ such that N units of D$p$brane charge cannot be localized to values smaller than r_c. At this value of r_c the curvature is of order $1/\alpha'$ and the gradient of the Ramond-Ramond field strength is of order $1/(\alpha')^{3/2}$, and the vacuum, regarded as a D-brane–anti-D-brane configuration *with* a tachyon condensate, is rendered unstable, leading to a separation of the D-brane and the anti-D-brane. This value of r_c lies in the region intermediate between the near-horizon regime and the asymptotic regime for D-brane classical solutions for small g. This vacuum stability bound on the curvature can be interpreted as an uncertainty relation for D-brane charge.

Keywords: D-branes, M(atrix) Theories.

Sen [1, 2] has recently found remarkable exact descriptions of D-brane–anti-D-brane configurations that exhibit crucial connections between non-BPS states in dual string theories. Witten [3] related these results to K–theory, putting the specific examples found by Sen [1, 2] into a general framework. In this paper, we examine a particular physical consequence of these ideas [1, 2, 3].

D-brane–anti-D-brane configurations are not supersymmetric by themselves. Indeed, they exhibit tachyonic instabilities for short distances [4, 5]. The particular aspect of Sen's work that will be of interest to us is that tachyon condensation leads to a stable configuration of a D-brane lying on top of an anti-D-brane [1, 2]. With such a tachyon condensate, Sen [1, 2] and Witten [3] have argued that this configuration is equivalent to the vacuum, hence it is supersymmetric in particular. This stable configuration of a D-brane–anti-D-brane pair with a tachyon condensate has the important property that the open string mode that corresponds to separating the D-brane from the anti-D-brane becomes massive. Tachyon condensation leading to stable configurations appeared also in [6, 7, 8].

In [2] Sen found an exact conformal field theory description of an anti-D-string–D-string pair with a $\mathbf{Z}_2$ Wilson line on one of the strings. This turned out, when the tachyon was given an expectation value, to be a configuration describing a particle, the strong coupling limit of the massive $\mathrm{Spin}(32)/\mathbf{Z}_2$ spinor state of the heterotic string. In particular, this configuration was shown to possess SO(9) invariance, which implies that, except at the position of the kink of the tachyon field produced by the presence of a Wilson line, along the anti-D-string–D-string pair, the configuration is equivalent to 'nothing', as mentioned above. See also the discussion in Witten [3]. Sen [2] also showed that the mode corresponding to relative motion of the D-string–anti-D-string acquires a mass when the tachyon condenses. This mode is a linear combination of the U(1) modes corresponding to motion of the D-string and the anti-D-string, and is not projected out by the GSO projection since it is a mode that corresponds to strings stretched from the D-string to itself, and strings stretched from the anti-D-string to itself.

We expect that these features should appear in any such Dpbrane–anti-Dpbrane configuration, independent of p. Specifically we assume that 'nothing' is equivalent to a D$p$brane–anti-D$p$brane with a tachyon condensate, and that modes corresponding to relative motion of the D$p$brane–anti-D$p$brane acquire a finite mass when the tachyon condenses. As indicated in the preceding paragraph, these facts were explicitly demonstrated by Sen for the D-string–anti-D-string configuration with a Wilson line, but the presence of the Wilson line does not seem to imply anything other than the formation of a kink in the tachyon condensate. From a spacetime perspective, it is difficult to see how either of our assumptions depends on the presence of the Wilson line. We are extrapolating from demonstrated facts [2] about D-strings to general D$p$branes. It could be that new features arise in general that invalidate our analysis.

In principle, the annulus diagram with the analogue of Sen's conformal field theory *without* the Wilson line should describe the force between D-branes and anti-D-branes in the presence of a tachyon condensate. However, we have not succeeded in eliminating the Wilson line from his construction [2] so we are forced to rely on a spacetime perspective for our conclusions. As we discuss below, we will only need a 'sub–stringy' domain where the forces due to supergraviton exchange are still small, so this spacetime perspective may be justified.

An important point: Our discussion will be phrased in terms of a radial coordinate r transverse to the branes. We will be computing gradients in the radial direction at such values of the radius that covariant derivatives can be replaced by partial r derivatives to lowest order in gN. Since the proper distance to the coordinate $r = 0$ is infinite for $p < 4$, it is not strictly correct in these cases to speak of the charge radius, since r_c is an infinite proper distance away from $r = 0$. The actual determination of r_c below will rely only on the magnitude of the gradient of the force felt by an anti–D-brane, and is therefore independent of the proper distance to $r = 0$.

A detailed analysis of the classical size of D-branes was given by Barbón [9] some time ago. It is informative to compare his discussion with that given below.

We consider such a 'probe' D-brane–anti-D-brane pair *with* a tachyon condensate in the background produced by a source of N D-branes. Remark that it is an abuse of common terminology to talk of this configuration as a 'probe' because it is equivalent to the vacuum [1, 2, 3] — what we are actually considering is the polarization of the vacuum produced by the N D-branes. Whenever we use the term 'probe' in the following, we always mean the brane–antibrane–tachyon condensate configuration equivalent to the vacuum. The probe D-brane feels no force due to the source because of its BPS character, but the probe anti-D-brane feels a static attraction towards the source. Since the open string mode corresponding to separating the probe pair is massive, this attraction merely gives this field in the world-volume theory of the probe pair an expectation value. However, because the source sets up a static force on the D-brane–anti-D-brane probe that has a non-vanishing gradient, it follows that at a critical value of the radial coordinate r_c between the probe and the source, the separation mode becomes massless and the anti-D-brane can combine with the source. The end result is a configuration in which the probe pair has been reduced to a probe D-brane, and the source configuration has $N - 1$ D-branes. Thus, r_c measures the critical value of the radial coordinate below which it is not possible to localize D-brane charge. The dependence of this critical value of r_c on the string coupling g will be deduced below.

The coupling of a D-brane to background fields is given by the Dirac–Born–Infeld action with a Chern–Simons term

$$S_p = -\mu_p \int \mathrm{d}^{p+1}\xi \mathrm{e}^{-\Phi} \Big[-\det G_{ab} + B_{ab} + 2\pi\alpha' F_{ab} \Big]^{1/2} + S_{cs}\,, \tag{1}$$

with

$$S_{cs} \equiv i\mu_p \int \left(\mathrm{e}^{B_{ab}+2\pi\alpha' F_{ab}} \wedge \sum_q C_q \right)_{p+1} , \tag{2}$$

in notation as in [10]. The metric and Ramond-Ramond fields produced by a source of N D$p$branes are

$$\mathrm{d}s^2 = f^{-1/2}\mathrm{d}x_{\parallel}^2 + f^{1/2}\mathrm{d}x_{\perp}^2 , \tag{3}$$

with a dilaton

$$\exp(-2\Phi) = f^{(p-3)/2} , \qquad \text{with} \qquad f \equiv 1 + \frac{NgK_p}{7-p}\frac{\alpha'^{(7-p)/2}}{r^{7-p}} \tag{4}$$

and an antisymmetric tensor field strength, obtained from a p-form potential $C_p \propto f^{-1}$.[1]

The appearance of a massive separation mode X implies that there is a term that should be added to eq. (1) of the form

$$\Delta S_p = \frac{1}{g}\int \mathrm{d}^{p+1}\xi \exp(-\Phi)\left[-\det G_{ab}\right]^{1/2} \frac{1}{4\alpha'}X^2 . \tag{5}$$

Now, for an anti-D-brane–D-brane pair, the complete potential consists of two terms, the potential felt by the anti-D-brane due to graviton-dilaton and Ramond-Ramond exchange and the mass term for the separation mode. The energy change if we displace the anti-D-brane relative to the D-brane in the direction of the source is

$$\partial_r f^{-1}\delta X\left(2 + \frac{1}{4\alpha'}X^2\right) + \frac{1}{2\alpha'}f^{-1}X\delta X . \tag{6}$$

Stability requires that the second variation with respect to X should be positive definite evaluated at $X = 0$. An elementary explicit calculation shows that at a critical radius r_c, X becomes massless when

$$\left(\frac{r_c}{\sqrt{\alpha'}}\right)^{9-p} \approx 4(8-p)K_p gN , \tag{7}$$

up to higher powers of gN. At this radius, the anti-D-brane is no longer bound to the D-brane, and can move away without incurring any cost in energy. Thus for $r < r_c$, anti-D-brane–D-brane pairs can be nucleated by the gradients of the background fields due to the source D-branes. This is *not* a non-perturbative process — it follows entirely from the statement that the D-brane–anti-D-brane configuration *with* a tachyon condensate is equivalent to 'nothing' [1, 3].

Let us evaluate the value of f at this value of r_c (eq. (7)): We find that $f(r_c) = 1$ up to terms of order $(gN)^{2/(9-p)}$. Thus the metric is close to the Minkowski metric as far as distance measurements are concerned in the vicinity of r_c. There is no physical significance to the value of r_c but computing the curvature (or the gradient of the Ramond-Ramond field strength) at r_c eq. (7) we find $R_{\mu\nu} \propto 1/\alpha'$. Thus the

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coordinate independent physical significance of this critical value of the coordinate r is that the curvature produced by the source D-branes becomes of order 1 in string units. $1/\alpha'$ appears here as a bound on curvatures produced by D-branes, which is quite different from the size of D-branes as determined by string scattering [11]. Our bound is, in fact, more reminiscent of an uncertainty relation, since the curvature is exactly the commutator of covariant derivatives — what we have found is that the vacuum is rendered unstable if there are Ramond-Ramond field strength gradients that are larger than $1/(\alpha')^{3/2}$. Such Ramond-Ramond field strengths are always accompanied by curvature, though the converse does not hold.

Since we have arrived at a conclusion that involves curvatures of order $1/\alpha'$, while taking into account only the massless closed string backgrounds induced by the N D-branes, we must address the question of massive closed string backgrounds. Recall that from a string field theory perspective, massive closed string modes when integrated out lead to the nonlinearities of supergravity. Inclusion of massive closed string modes is therefore related to including nonlinear effects in supergravity. The coordinate r_c lies in exactly the intermediate regime between the near–horizon geometry and the asymptotic geometry. The latter is adequately described by single supergraviton exchange, and massive string backgrounds are not induced in this regime. Now, one can compute the force felt by the anti-D-brane in the intermediate regime at r_c, and one finds that this force is small, of order $(gN)^{1/(9-p)}$, at r_c.

While pure supergravity is only accurate for $r > r_c$, the supergravity calculation should still be a valid qualitative guide to the behaviour at r_c, without including either nonlinearities or massive closed string modes.

Notice that r_c scales with N for all p as if the N units of charge were distributed uniformly over a transverse sphere of radius r_c (as measured in a flat metric). This extensivity suggests that our conclusions may be valid for N small as well, though the discussion above assumes N large. Measured in a flat metric, for $p = 0, N = 1$ this critical radius $r_c \propto \sqrt{\alpha'} g^{1/9}$ should be compared to $\ell_{11} \propto \sqrt{\alpha'} g^{1/3}$, the Planck length in eleven dimensions, which appears in D0brane quantum mechanics [13, 14, 15]. For small g r_c is much larger than ℓ_{11}.

Considering the N dependence of r_c around a configuration of N D0branes, we find $r_c \propto N^{1/9}$. Recall that in [12] such an N dependence of length scales was argued to be required for holography on rather general grounds. There are no anti–D0branes in the light–front Hamiltonian [12], but the N dependence of r_c that we find suggests that the physics of a finite charge radius could be incorporated in the mysterious vacuum state of the light–front Hamiltonian.

For $p = 3$ and small gN, $r_c \propto \sqrt{\alpha'}(gN)^{1/6} \gg \sqrt{\alpha'}(gN)^{1/4} \propto R$, the length scale characterizing the geometry at small values of r. Thus, at small gN, we would expect the IIB string theory to exhibit effects of a finite charge 'radius'. Since our estimate is not valid for large gN, we cannot extrapolate to this regime, but clearly one would expect a qualitative change in the nature of the physics when $r_c \approx R$, i.e. at $gN \approx 1$.

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This is also the length scale at which stringy effects should become important in Maldacena's conjecture [16]. How the physics of r_c would appear in a sigma model with the isometry group appropriate for the near–horizon geometry is an interesting question.

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